# THE COSTS OF WHEELING AND OPTIMAL WHEELING RATES

Michael C. Caramanis
Boston University
College of Engineering
(Member)

Roger E. Bohn
Harvard University
Graduate School of Business Administration
(Member)

Fred C. Schweppe
Massachusetts Institute of Technology
Laboratory for Electromagnetic and Electronic Systems
(Fellow)

Abstract

Wheeling is the transmission of electrical energy from a buyer to a seller through a transmission network owned by a third party. This paper provides a theoretically sound, yet practical to implement, basis for setting wheeling rates. These wheeling rates are based on marginal costs (determined by losses and effects of line flow and voltage magnitude constraints) adjusted up or down as necessary to account for embedded capital costs (i.e. revenue reconciliation). Simple numerical examples are provided to illustrate interesting phenomena such as negative wheeling rates which yield positive net revenue. Comparisons with present day wheeling rates show that major differences exist. The wheeling rates of this paper yield a "no lose" situation for the buying, selling and wheeling utilities.

## Section 1 Introduction

Wheeling is the transmission of electrical energy from a buyer to a seller, through transmission or distribution lines owned by a third party. It is becoming of increasing importance in the operation and planning of the U.S. power system. Wheeling rates determine payments by the buyers or sellers (or both) to the wheeling utility to compensate the wheeling utility for the generation and network costs it incurs.

Present day practice on wheeling rates is quite varied, and has evolved in response to specific situations. The purpose of this paper is to provide a coherent, consistent engineering and economic basis for setting wheeling rates. Our approach provides a starting point for rate setting or negotiations in any situation.

Some of our results are quite new and different from conventional wheeling practice. For example our wheeling rates can sometimes be negative and are not necessarily related to the distances involved. On the other hand, our approach can be viewed as a relatively straight forward extension of certain present day techniques of power system operation.

The Physics and Engineering of Wheeling: To understand wheeling, it is necessary to understand how electricity flows in A.C. transmission networks. Call the selling utility Utility S, and the buying utility Utility B. Suppose they are non-contiguous, and several parallel pathways through different utilities connect them. One of the connecting utilities is Utility K. What does it mean to say that Utility S is wheeling to Utility B, via Utility K?

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One thing this does not mean is that a specific identifiable set of electrons or electrical energy is "shipped" from S to B, over the lines of K. Electrical flows cannot be directed from one specific point to another specific point. Rather, all generators inject energy into the transmission grid, and all customers remove energy from the grid.

The actual process is as follows. If Utility S and Utility B decide to engage in a new transaction involving W MWH of energy during hour t, Utility S increases its net scheduled interchange (a parameter of its automatic generation control (AGC) system) by W while Utility B decreases its net scheduled interchange by W. This changes the flows throughout the network, including the lines of Utility K. The specific flow changes are not under the direct control of any of the utilities. Kirchoff's laws and the changed generation patterns determine what happens. Some of this W MWH of energy will flow through Utility K independent of whether or not Utility K gives its permission. The sum of all tie line flows into and out of Utility K do not change. Utility K's costs are affected because of changes in its internal losses (which affect its generation costs) and possible impacts on line flow and voltage magnitude constraints. Furthermore the costs of the capital Utility K has invested in its system cannot be ignored.

Criteria: An ideal system of wheeling rates would satisfy a number of criteria such as:

- Cause buying and selling utilities to make "efficient" wheeling decisions based on operating costs, embedded capital, and power system security.
- Enable multiple wheeling transactions to occur simultaneously.
- Provide incentives to utilities to strengthen their transmission systems when mutually beneficial to all utilities.
- Allow for as much decentralized decision making as possible while providing a range of consistent options which match transactions costs with benefits.
- Reduce as much as possible the opportunity for arbitrary or political decisions on wheeling rates.
- Be feasible to calculate and implement.

The rates developed here fulfill all of these criteria fairly well.

Outline: In Section 2 we develop the marginal dollar cost to Utility K of wheeling. In Section 3 we develop two simple network examples in detail. In Section 4 we discuss revenue reconciliation. Section 5 covers practical calculation of our rates; what information is required, and when it is required. Section 6 discusses market arrangements for wheeling. Section 7 looks at present wheeling rates and compares them with our approach. Section 8 is a summary.

This paper addresses wheeling between regulated, independent utilities. Most of the basic ideas and principles also apply to other cases such as when an industrial cogenerator wants to sell energy directly to another industrial company with the local utility doing the wheeling. However, we do not explicitly address such other cases in this paper.

We have attempted to write this paper so that the general ideas can be understood by readers without a strong background in power system operations. However, such a background is needed to understand some of the more technical issues. Many papers and books have been written on the subject. One excellent example is Ref. [1].

## Section 2 Spatial Spot Pricing and Wheeling Rates

The foundation for our analysis is the theory of spot pricing [Ref 2,3,4,5]. As demand patterns, generator availability, and transmission availability vary over a day and over longer periods, the utilities' dispatching patterns and costs vary. This gives rise to time varying spatial spot prices, which capture all relevant economic and engineering information. Because of losses and system security (line flow) limits, a kwh of energy has different values at different buses of the network. Since wheeling is analagous to buying energy at one set of buses (those tie line buses with incremental flows into K) and selling it at another set (those incrementally flowing out of K), these spatial price differences determine the cost of wheeling.

Our approach defines wheeling rates in terms of marginal (incremental) costs which are adjusted up or down if necessary to assure revenue reconciliation. (I.e. recovery of capital and operating costs.) However, we deter consideration of revenue reconciliation until Section 4.

Deriving Wheeling Rates: Consider a system of N interconnected utilities. Define

$W_{BS}(t)$: Energy Utility S is selling to Utility B during hour t

$\underline{T}^K(t)$: Vector of energy flows during hour t over tie lines of Utility K.

$I^B(t)$, $I^S(t)$: Scheduled Net Interchange of Utilities B and S during hour t.

$C^K(t)$: Cost incurred by Utility K during hour t ($).

$\omega_{K,BS}(t)$: Optimum wheeling rates during hour t if revenue reconciliation is ignored ($/kwh).

We define the optimal wheeling rate as the marginal impact of wheeling on Utility K's costs;

$$\omega_{K,BS}(t) = \frac{\partial C^K(t)}{\partial I^B(t)} \quad (\$/kwh) \tag{2.1}$$

subject to the conditions

- o An incremental change in Utility B's scheduled net interchange is matched by an equal but opposite change by Utility S;
- o Scheduled net interchange of all other interconnected utilities are held constant. (2.2)

The Automatic Generation Control (AGC) systems of all interconnected utilities are well defined algorithms which maintain the above conditions at minimum cost for each utility. In this context the AGC systems determine, other things being equal, all internal parameters (generation levels, marginal generator locations, etc.) as a function of tie line flows. Therefore, considering all internal parameters as dependent upon tie line flows, Eq. (2.1) can be written as

$$\omega_{K,BS}(t) = [\underline{\rho}^K(t)]' \, [\underline{D}_{K,BS}(t)] \tag{2.3}$$

$\underline{\rho}^K(t)$: Vector of Utility K's tie line bus spot prices.

$$\underline{\rho}^K(t) = \frac{\partial C^K(t)}{\partial \underline{T}^K(t)}$$

$\underline{D}_{K,BS}(t)$: Vector of tie line flow coefficients

$$\underline{D}_{K,BS}(t) = \frac{\partial \underline{T}^K(t)}{\partial I^B(t)}$$

where ' denotes vector transpose and the derivatives are all evaluated subject to the conditions of Eq. (2.2). In general the tie line flow coefficients and tie line bus spot prices depend on the behavior of the AGC systems of all N utilities, i.e. on which generators are marginal, etc. The use of a one hour interval as the basic time unit is of course not essential.

Discussion of Tie Line Flow Coefficients: The tie line flow coefficients give the impact of wheeling changes on tie line flows. There is a different vector of tie line flow coefficients for every combination of wheeling and trading utilities. Negative tie line flow coefficients correspond to tie lines whose flow into K increases with $W_{BS}(t)$ ("Marginal inflow buses") But note that this refers to the marginal flows; the actual flow on a tie line may be in the opposite direction. Positive coefficients correspond to tie lines whose flow out of K increases with $W_{BS}(t)$ ("Marginal outflow buses"). In the special three utility world of Figure 1, all the energy is wheeled through Utility K, the tie line flow coefficient is -1 for the tie line into K from S, and +1 for the tie line from K to B.

The sum of the elements of the tie line flow coefficients must be 0. Thus Eq. 2.3 is the difference between spot prices at the marginal outflow buses and the marginal inflow buses, weighted by the absolute values of the tie line flow coefficients at each bus. In the special case of Figure 1, $\omega_{K,BS}(t)$ is the difference between the spot prices at the two buses.

2 BUS EXAMPLE: REPRESENTATION OF WHEELING UTILITY

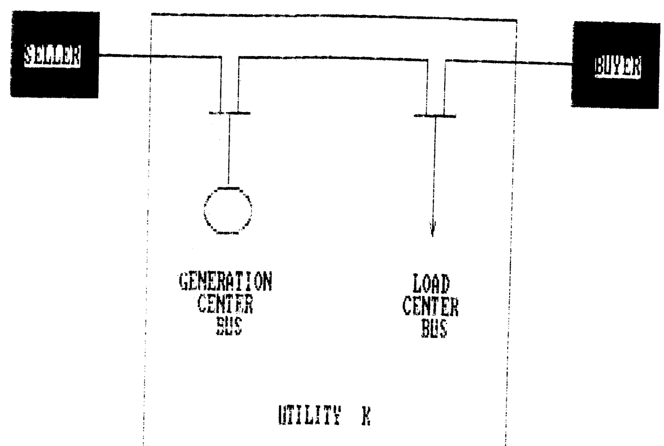



FIGURE 1.

Discussion of Spatial Spot Prices: From Ref. [2] we have the following equation for optimal spot prices at bus i, during hour t.

Optimal spot price at bus i = [cost of additional demand at the swing bus] x [1 + incremental losses caused by i] + [transmission constraint terms, summed over lines]

$$\rho_i(t) = \Theta(t)\ [1 + \frac{\partial L}{\partial d_i(t)}] + \sum_j \frac{\partial Z_j(t)}{\partial d_j(t)} \eta_j(t) \quad (2.4)$$

where

$\rho_i(t)$ = Optimal spot price at bus i,

$L(t)$ = Transmission losses

$d_i(t)$ = Demand at bus i (negative number if generation)

$Z_j(t)$ = Line flow in line j

$\eta_j(t)$ = Opportunity cost of transmission limits on line j

= 0 when line j not fully loaded.

$\theta(t) = \lambda(t) + \mu(t)$

$\lambda(t)$ = System lambda for Utility K

$\mu(t)$ = 0 except when demand is approaching its available generation limit.

The $\eta_j(t)$ and $\mu(t)$ yield the generation and network "quality of supply" components of the spot price. They can be viewed as "system security" prices.

When neither generation nor line limits are binding, the wheeling rate $\omega_{K,BS}(t)$ in Eq. (2.3) is approximately the weighted sum of the difference in system marginal losses at the marginal inflow buses and the marginal outflow buses, multiplied by system lambda. The weights are provided by the tie line flow coefficients.

The rate $\omega_{K,BS}(t)$ for wheeling from S to B will:

- Increase if Utility K's losses increase, and decrease (going negative) if K's losses decrease.
- Increase or decrease if Utility K's transmission network flow and voltages are close to their limits, as they change in a direction moving closer to or away from such limits.
- Increase in magnitude as Utility K's generation costs increase due to high customer demand, generation outages or hourly sales to other utilities.

Determining Wheeling Charges: The basic formula for wheeling rates is Eq. 2.3. This means for hour t

$$\text{Total payment from B,S to K} = W_{BS}(t)\,\omega_{K,BS}(t) \quad (2.5)$$

$$\text{Total payment from B,S to all N connected utilities for wheeling} = \sum_K \omega_{K,BS}(t)\, W_{BS}(t) \quad (2.6)$$

$$\text{Total Payment to K by all utilities which are wheeling} = \sum_B \sum_S \omega_{K,BS}(t)\ W_{BS}(t) \quad (2.7)$$

Net Revenues from Wheeling: A dollar value of considerable interest is the net revenue to utility K from wheeling. Gross revenue, given by Eq. (2.5) for one pair of trading utilities and Eq. (2.7) for all trading utilities, can be negative. Define

$$\begin{pmatrix}\text{Net}\\ \text{Revenue}\\ \text{from Wheeling}\end{pmatrix} = \begin{pmatrix}\text{Gross}\\ \text{Revenue}\\ \text{from Wheeling}\end{pmatrix} - \begin{pmatrix}\text{Changes in Total}\\ \text{Costs to K due to}\\ \text{Wheeling}\end{pmatrix} \quad (2.8)$$

Net revenue to K is always positive even if gross revenue is negative. This is because the wheeling rate $\omega_{K,BS}(t)$ is at Utility K's marginal costs which is close to its average cost when $W_{BS}(t)$ is small, but it exceeds the average cost as $W_{BS}(t)$ becomes large. Section 3 presents numerical examples of this effect which is due in part to the nonlinear nature of line losses.

Net revenue of S and B from wheeling is also positive to the extent they are engaging in economy interchange or the equivalent, i.e. they are using their own instantaneous spot prices to decide whether or not to interchange.

Reactive Energy and Voltage Magnitudes: The preceding discussions have considered only real energy. Reactive energy flows can also be important as they affect both real line losses and voltage magnitudes. In practice, it would sometimes be desirable to include a wheeling rate on reactive energy flows. The preceding equations can be generalized by viewing the energies and prices as complex numbers whose real and imaginary parts correspond to real and reactive energy respectively. One significant difference is that present day AGC systems do not control the sum of reactive flows entering and leaving a utility. Spot prices for reactive energy flows are derived in Ref. [2].

## Section 3 Numerical Examples

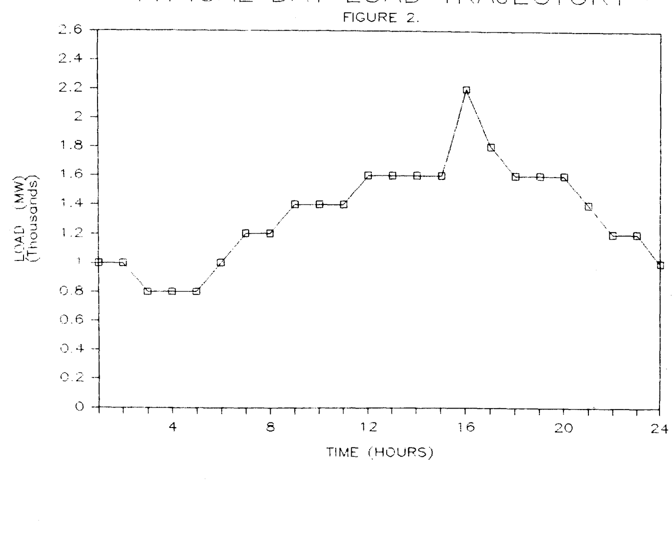
Two "textbook" examples are employed to illustrate the behavior of optimal wheeling rates. In the two bus example we look at the direction of power flows, and show how negative wheeling rates can arise. In the three bus example we add transmission line constraints.

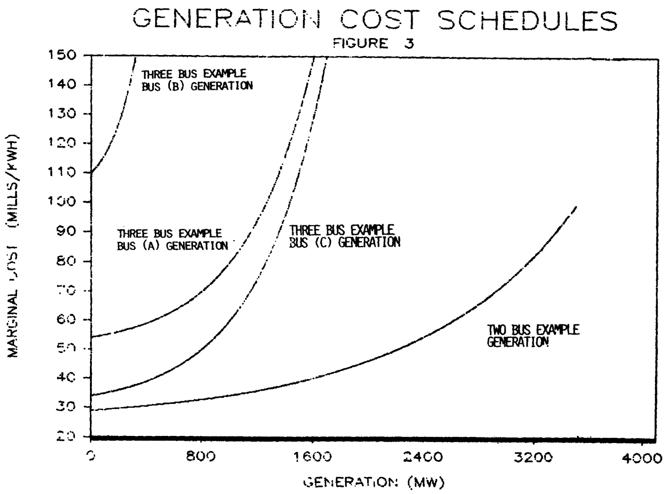
### Section 3.1 The Two Bus Example

Assumptions: In the two bus example, the wheeling utility, Utility K, is approximated by a "load center" bus and a "generation center" bus connected by a single transmission line as depicted in Figure 1. Load during a typical day is assumed to vary between 800 and 2200 MW according to the hourly trajectory of Figure 2. Transmission losses range from 0.8% at minimum load to 2.2% at peak load and are modeled as a quadratic function of transmission flows. Marginal variable generation cost varies continuously from 35 to 70 mills/kwh as shown in Figure 3.

TYPICAL DAY LOAD TRAJECTORY

FIGURE 2.






<u>Optimal Wheeling Rate Behavior</u>: Two cases considered are:

Case 1: Utility S (selling) injects energy at Utility K's generation bus while Utility B (buying) extracts the same energy from K's load bus; thereby increasing losses.

Case 2: Positions of Utilities S and B are reversed so that transmission losses decrease with wheeling.

For this special case, Eq. (2.3) and (2.4) yield (subscripts K, B and S are not shown)

$$\omega(t) = 2\lambda(t)B[d(t) + W(t)] \text{ in Case 1} \qquad (3.1)$$

$$\omega(t) = -2\lambda(t)B[d(t) - W(t)] \text{ in Case 2} \qquad (3.2)$$

where the generation center is designated as the swing bus, $\lambda(t)$ is the marginal cost of generation during hour t (system lambda), B the quadratic loss coefficient (set equal to $10^{-5}$ on a MVA unit basis), d(t) Utility K's load during hour t, and W(t) the amount of traded energy during hour t.

Case 1 which increases Utility K's transmission losses yields positive wheeling rates that increase as Utility K's load level or the amount of wheeling increases, as shown in Eq. (3.1). Table 1 reports optimal wheeling rates for Case 1. For a low level of wheeling (W = 1 MWH) wheeling rates vary from 0.5 mills/kwh at minimum load to 2.25 mills/kwh at peak load. For a higher level of wheeling, (W = 401), wheeling rates vary from 0.79 to 2.68 mills/kwh. This is because the direction of wheeling is the same as that of power flows for serving load.

<u>Table 1</u>

<u>Two Bus Example: Seller Close to Generation Center</u>

| Load MW | Wheeling Rate W=1 | W=201 | W=401 | Net Wheeling Revenue W=1 | W=201 | W=401 |
|---|---|---|---|---|---|---|
| 800 | 0.53 | 0.66 | 0.79 | .0003 | .066 | .132 |
| 1000 | 0.69 | 0.82 | 0.96 | .0003 | .069 | .138 |
| 1200 | 0.86 | 1.01 | 1.16 | .0004 | .073 | .146 |
| 1400 | 1.07 | 1.22 | 1.38 | .0004 | .076 | .155 |
| 1600 | 1.30 | 1.46 | 1.63 | .0004 | .083 | .167 |
| 1800 | 1.56 | 1.74 | 1.92 | .0004 | .089 | .180 |
| 2000 | 1.88 | 2.07 | 2.27 | .0005 | .098 | .197 |
| 2200 | 2.25 | 2.46 | 2.68 | .0005 | .108 | .217 |

Units: mills/kwh

In Case 2, wheeling decreases Utility K's transmission losses as reflected by the negative sign in Eq. (3.2). Negative wheeling rates in effect distribute the benefit from reduced losses between the wheeling and the trading parties. Table 2 reports optimal wheeling rates for Case 2. At minimum load and low wheeling power level, (W = 1), negative optimal wheeling rates indicate that Utility K should pay the trading parties 0.53 mills/kwh, with the payment increasing to 2.25 mills/kwh at peak load. This is due to the fact that the direction of wheeling is now opposite to the direction of power flows for serving load. Note, however, that although the marginal decrease in transmission losses (i.e., savings due to wheeling) is proportional to load, it is a decreasing function of the amount of wheeled power. Thus, the corresponding payments to the trading parties for the high wheeling power level, (W=401), are 0.39 mills/kwh) at minimum load and 1.82 mills/kwh at peak load.

<u>Table 2</u>

<u>Two Bus Example: Seller Close to Load Center</u>

| Load MW | Wheeling Rate W=1 | W=201 | W=401 | Net Wheeling Revenue W=1 | W=201 | W=401 |
|---|---|---|---|---|---|---|
| 800 | -0.53 | -0.39 | -0.26 | .0003 | .066 | .132 |
| 1000 | -0.69 | -0.55 | -0.41 | .0003 | .069 | .138 |
| 1200 | -0.86 | -0.72 | -0.57 | .0004 | .073 | .145 |
| 1400 | -1.07 | -0.91 | -0.76 | .0004 | .077 | .153 |
| 1600 | -1.30 | -1.13 | -0.97 | .0004 | .082 | .164 |
| 1800 | -1.56 | -1.39 | -1.21 | .0004 | .089 | .176 |
| 2000 | -1.88 | -1.68 | -1.49 | .0005 | .097 | .192 |
| 2200 | -2.25 | -2.03 | -1.82 | .0005 | .106 | .210 |

Units: mills/kwh

<u>Utility K's Net Revenue</u>: Tables 1 and 2 report the net revenue per kwh of energy traded. The net wheeling revenue for Utility K is positive even when the wheeling rate and hence the gross wheeling revenue are negative. As discussed in Section 2, this is a general result that does not depend on the two bus approximation employed here. The positive net revenues can be seen in the last three columns of Tables 1 and 2 to vary from .0003 mills/kwh or practically zero at W = 1 and minimum load to over 0.2 mills/kwh at W = 401 and peak load, in both the positive and negative wheeling rate cases.

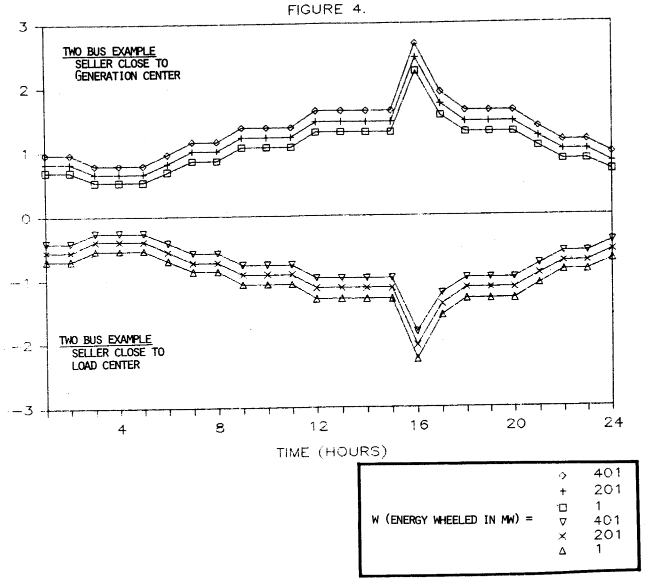
<u>Wheeling Rate Time Trajectories</u>: Optimal wheeling rates vary over time as load, generation and transmission line conditions change. If, as discussed further in Section 5, it is desirable to base transactions on a fixed rate to decrease transaction and communication costs, the time varying optimal wheeling rates can be used for a consistent determination of the fixed rates. The hourly trajectories of optimal wheeling rates corresponding to the load trajectory of Figure 2 are shown for three levels of wheeled power for Cases 1 and 2 in Figure 4. The time averaged values are somewhat larger than 1 mill/kwh in Case 1, while for Case 2 they are between -0.7 and -1.0 mills/kwh.

<u>Section 3.2 Three Bus Example</u>

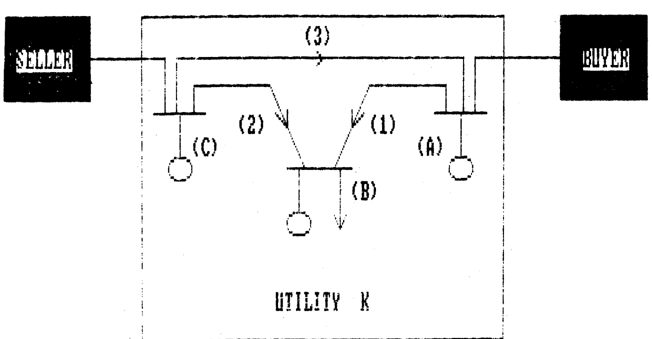
<u>Assumptions</u>: The "three bus" example is employed to illustrate the effect of transmission line capacity constraints on optimal wheeling rates. The three bus - three line model used to approximate Utility K is shown in Figure 5. Buses (a) and (c) are generation centers with variable generation costs under normal outage conditions ranging respectively from 55 and 35 mills/kwh at low generation levels, to 80 and 60

TWO BUS WHEELING PATE TRAJECTORIES

FIGURE 4.




3 BUS EXAMPLE: REPRESENTATION OF WHEELING UTILITY




NOTE:
Most Expensive Generation at Bus (B)
Least Expensive Generation at Bus (C)
Transmission Line (2) Capacity Limited to 1000 MW

FIGURE 5.

mills/kwh at generation levels of 1000 MW as shown in Figure 3. Bus (b) is modeled as having some expensive generation (variable costs exceed 110 mills/kwh at normal outage conditions) and all of Utility K's load. To examine the likely impact of high generation outages on wheeling rates, generation costs for "high outage conditions" are also used later.

All three transmission lines in Figure 5 are assumed to be identical with a resistance of 0.04 on a GVA unit basis. Power flows and transmission losses are modeled using a DC load flow model with bus (c) designated as the swing bus (See Appendix A). Losses vary with load from 2% to 4%. Power flow through transmission line (2) is limited to a maximum of 1000 MW. Finally, a wheeled energy level of 200 MWH is used throughout the three bus example.

Optimal Wheeling Rate Behavior: Table 3 shows optimal wheeling rates for different load levels of Utility K. While transmission line capacity constraints are not binding, wheeling rates do not exceed 2 mills/kwh. When they become effective the optimal wheeling rate increases to as much as 36 mills/kwh. This reflects the high costs incurred by Utility K in rearranging its generation pattern toward higher generation at buses (a) and (b) and lower generation at bus (c), in order to meet the transmission line (2) capacity constraint. The extent to which the wheeling utility has to rearrange its generation pattern to meet the flow constraint can be appreciated by observing the last column of Table 3 which shows line (2) flows without generation redispatch.

Table 3

Three Bus Example: Transmission Line 2 Capacity Limited

| Load MW | Wheeling Rate (mills/kwh) | Line (2) Power Flow Without Redispatch* (MW) |
|---|---|---|
| 800 | 1.62 | 600 |
| 1000 | 1.60 | 687 |
| 1200 | 1.48 | 766 |
| 1400 | 1.37 | 847 |
| 1600 | 1.30 | 930 |
| 1800 | 6.02 | 1014* |
| 2000 | 27.00 | 1101* |
| 2200 | 36.10 | 1189* |

*Generation must be redispatched for load levels 1800, 2000 and 2200 in order to meet transmission line (2) maximum flow constraint of 1000 MW.

A difference between the two examples considered so far should be noted. Wheeling rates are not as simply correlated to load in the three bus example as they are in the two bus example for two reasons. First wheeling is in the same direction as flows over lines (2) and (3) but in the opposite direction as flows over line (1) as can be seen from Figure 5. Second is the effect of the system security constraint on line (2).

It should be noted that interchanging the location of buying and selling utilities in the three bus example would yield negative wheeling rates just as in the two bus example. However, net wheeling revenues would always be positive.

Wheeling Rate Time Trajectory: Using the hourly load trajectory of Figure 2, the wheeling rate hourly trajectory for the three bus example is obtained in Figure 6. It exhibits a much larger swing of values than in the two bus example. The time averaged wheeling rate is 3 mills/kwh. Note that despite the relatively low average wheeling rate, optimal time varying wheeling rates would provide an incentive for reduced trade during hours 16 and 17 when line flow constraints are binding. If no wheeling took place during these hours, the average wheeling rate would decrease from 3 to less than 1.5 mills/kwh.

THREE BUS WHEELING RATE TRAJECTORY

UNDER NORMAL OUTAGE CONDITIONS

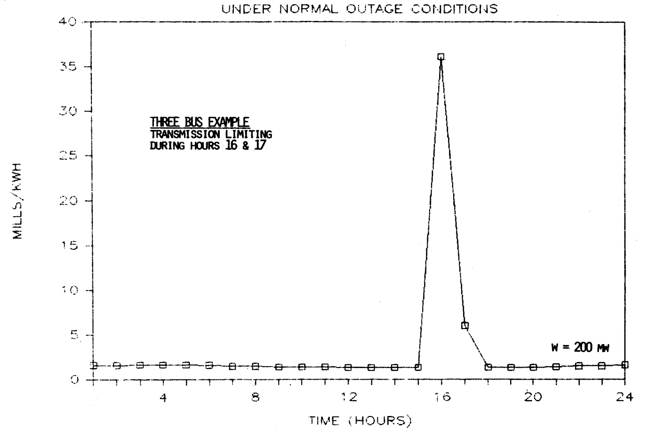



FIGURE 6

Generation Outages: The numerical examples presented so far have focused on the variation of one parameter, namely customer load. However, wheeling costs are also sensitive to changes in generation and transmission line availability. To provide some insight on the impact of such changes, the wheeling rate trajectory of Figure 6 was recomputed with higher generation costs at bus (b) which reach 240 mill/kwh at 50 MW to reflect unusually high forced or planned outages of generating units located at the load bus (b). The high outage trajectory is similar in appearance to Figure 6 but the wheeling rates range from 1.3 to as high as 83 mills/kwh. Thus if the information contained in the hourly trajectory were seen by the trading utilities it would be very likely to affect their wheeling decisions. Indeed if the trading parties were to reduce the level of wheeled energy from 200 MWH to 20 MWH the wheeling rate would decrease from 83 mills/kwh to 53 mills/kwh during hour 17. The above numerical results are particularly relevant to the subsequent discussion in Section 5 on the implementation of time varying wheeling rates as opposed to predetermined "time of use" wheeling rates. If load variation is sufficiently predictable, its impact can be captured by "time of use" wheeling rates. Generation and transmission line availability, i.e. system security limits, however, are quite unpredictable.

Extensions: The preceeding discussions on the hourly variation of wheeling rates can be extended to examine seasonal or annual variations that may be simulated for planning purposes using load duration curves and described using "optimal wheeling rate" duration curves (see Ref [4] and Ref. [5].

## Section 4 Revenue Reconciliation

We now discuss revenue reconciliation, i.e., the impacts of embedded capital costs, rate of return on investment, etc., on wheeling rates. The wheeling utility is viewed as a regulated firm engaged in two different types of transactions, selling electric energy to customers in its own service territory and wheeling electric energy. (The case where the utility is also buying energy from customers such as cogenerators is closely related but is not discussed here).

The wheeling rates of Section 2, as illustrated in Section 3, always yield a positive net revenue to the wheeling Utility K (even if the rates themselves are negative). If Utility K's capital mix is "optimum", this net revenue matches Utility K's capital costs and revenue reconciliation occurs automatically. Revenue reconciliation is an issue because, in practice, a utility can either over recover or under recover using the Section 2 wheeling rates. Thus it is necessary to adjust the wheeling rates of Section 2, either up or down, depending on the utility's capital investments. Demand charges are neither needed nor desirable.

Under certain operating conditions, Utility K could readjust its own generation pattern and network configuration to yield a wheeling rate which artificially raises its own net revenue; i.e., it can exercise a type of monopolistic behavior. Revenue reconciliation reduces the incentive for such opportunistic behavior.

Define for Utility K:

- $R_c$: Annual revenues received by sale of energy to own customers. ($)
- $R_w$: Annual revenues received from wheeling rates. ($)
- $R_T$: Total annual costs for fuel, maintenance, operations, embedded capital costs, and allowed rate of return on investment. ($)

Revenue reconciliation is said to occur if the rates to Utility K's own customers and the wheeling rates are both adjusted such that

$$R_T = R_c + R_w \tag{4.1}$$

where

$$R_w = \sum_{t=1}^{8760} \omega(t)\, W(t) \tag{4.2}$$

$$R_c = \sum_{t=1}^{8760} \sum_{j=1}^{J} r_j(t)\, d_j(t) \tag{4.3}$$

$r_j(t)$: Rate ($/kwh) for jth customer of Utility K

$\omega(t)$: Wheeling rate ($/kwh) after revenue reconciliation adjustment

$d_j(t)$: Demand of jth customer of Utility K (kwh)

$W(t)$: Amount wheeled (kwh)

(The K and BS subscripts are dropped for convenience.) If many different buying and selling utilities are wheeling through Utility K, Eq. (4.2) for $R_w$ becomes a sum over all such buying and selling transactions. If Utility K is itself buying and/or selling with other utilities, Eq. (4.1) is modified to include such revenues or costs.

The basic principle discussed here is to choose $\omega(t)$ to be as close as possible to $\omega^*(t)$ subject to the constraint of Eq (4.1) where $\omega^*(t)$ is the wheeling rate without revenue reconciliation, i.e. the value given by Eq. (2.3). Of course there are many different ways to implement this principle. For example, since $\omega^*(t)$ depends on the tie line bus spot prices $\underline{\rho}^K(t)$ via Eq. (2.3), one could either try to match

$$\omega(t) \text{ to } \omega^*(t) \tag{4.4}$$

or alternately try to match

$$\underline{\rho}^K(t) \text{ to } \underline{\rho}^{K*}(t) \tag{4.5}$$

and then compute $\omega(t)$ from $\underline{\rho}^K(t)$. As an example of another variation, one can either measure distance in a weighted least squares sense or use a social welfare cost minimization philosophy. If a weighted least squares fit is used, there is freedom in the choice of weights. An example of a third variation is whether or not the same method of revenue reconciliation is applied to wheeling rates and to Utility K's own customer rates.

To be more specific, it is shown in Appendix B that when Eq. (4.4) is combined with weighted least squares using reasonable weights,

$$\omega(t) = \omega^*(t) + \mu_R\, |\omega^*(t)| \tag{4.6}$$

where $\mu_R$ is a Lagrange multiplier which is adjusted so that Eq. (4.1) is satisfied assuming both Utility K's own customers and the Utilities B and S share in the revenue reconciliation equally. The multiplier $\mu_R$ will be positive if Utility K would under recover if the marginal costs of Section 2 were charged and will be negative if Utility K would over recover. We are not necessarily advocating the use of Eq. (4.6). It is presented to illustrate one reasonable approach.

Separate Generation and Transmission Revenue Reconciliation: Eq. (4.6) achieves revenue reconciliation when all of the generation and network costs are combined together and the reconciliation is done simultaneously for both. However, it is possible to separate the revenue reconciliation into two effects, those arising from generation, and those from the transmission distribution network. This makes it possible to ask questions such as

- o Should revenue reconcilation for wheeling rates consider only network costs or should it also include generation costs?

We will not address this question in this paper but note that the answer can have a major impact on the wheeling rates. One approach for decomposing revenue reconciliation into two components is to view Utility K's generation system as "selling energy" to Utility K's transmission distribution network via a hypothetical set of transactions which are subject to revenue reconciliation for generation costs alone. Utility K's network is then viewed as wheeling energy to its own customers and wheeling for other utilities where both these transactions are subject to revenue reconciliation for network costs alone.

The above discussions illustrate the fact that revenue reconciliation can be done in a variety of ways. With certain approaches, it is mathematically possible for the net revenue of Eq (2.8) to become negative.

We believe that the choice of an explicit revenue reconciliation procedure from among the many available is a policy decision to be made by utility executives and/or regulatory commissions. However this does not mean that we believe revenue reconciliation should be allowed to become an arbitrary tool to be used for political purposes wherein the method changes with time depending on the desires of those with most influence at the given time. The choice of a single revenue reconciliation approach should be made once and applied equally to all wheeling and customer rates for all future times. Once this philosophical policy decision is made, the wheeling rates become uniquely specified by the costs. If revenue reconciliation is small (in magnitude), the choice of approach will not be critical.

## Section 5 Calculation of Wheeling Rates

Wheeling rate formulas have been presented and simple examples discussed to illustrate their behavior. Calculation of wheeling rates on a large system are now discussed.

One key implementation decision is whether to use both real and reactive energy rates or to restrict the wheeling rates to only real energy transfers. It is computationally easier to ignore the reactive energy, but in some cases it could be a very important component of the wheeling charges; e.g., when the ability to maintain voltage magnitudes within the wheeling utility is limiting. Both cases are discussed.

Four different types of wheeling rates are:

- o 1 Hour Update: Varies each hour. Based on predictions of conditions for next hour.
- o 24 Hour Update: Varies each hour. Made available 24 hours in advance based on predictions of conditions for next day.
- o Time of Use (TOU): Varies with time of day, day of week, season of year. Prespecified one year in advance based on expected conditions for next year.
- o Flat: Constant. Prespecified one year in advance, based on expected conditions.

These are analogous to the different types of spot prices proposed for a utility's own customers in Ref. [2]. The 1 and 24 hour update rates are "on-line" rates while the TOU and flat rates are "off-line" rates.

It follows from Eq. (2.3) and Eq. (4.6)

$$\begin{pmatrix}\text{Wheeling Rate}\\ \text{at hour } t\end{pmatrix} = \begin{pmatrix}\text{Tie Line Flow}\\ \text{Coefficients}\\ \text{at hour } t\end{pmatrix} \times \begin{pmatrix}\text{Tie Line Bus}\\ \text{Spot Prices}\\ \text{at hour } t\end{pmatrix} + \begin{pmatrix}\text{Revenue}\\ \text{Reconciliation}\\ \text{Component}\end{pmatrix} \tag{5.1}$$

Calculation of flow coefficients, bus spot prices and revenue reconciliation components are separate operations with different characteristics. Therefore they are discussed separately.

Tie Line Flow Coefficients: Given an AC load flow program (or DC load flow if reactive energy is ignored) covering all N utilities, it is "conceptually easy" to compute the tie line flow coefficients although some modification of a basic load flow code will usually be required. If a metering, communication and central computer system which gathers on-line network data from all N utilities exists (as in a sophisticated power pool), 1 or 24 hour update tie line flow coefficients could be computed. However, in general, such an on-line centralized facility will not be available. (This will change in future years as the level of communication and computation between interconnected utilities is continuously improving.) In such cases, the tie line flow coefficients have to be precomputed off-line as TOU or flat coefficients (or alternately prespecified for various overall demand levels). This off-line computation still requires a single AC (or DC) load flow program covering all N utilities. If such a program (i.e., data base) is not available, a committee of transmission engineers from the utilities could sit down and arrive at a consensus of what the tie line flow coefficients should be, based on their own experiences and professional judgement.

Tie Line Bus Spot Prices: Calculation of tie line bus spot prices are primarily local computations done by each wheeling utility (i.e., Utility K) without the need for extensive data transfer between all N utilities.[1] Various levels of sophistication can be used. System lambda as computed in the economic dispatch can be used directly, the effects of unit commitment could be added, lambda could be the average of incremental and decremental costs over some range of transactions, etc. The generation quality of supply components of the spot prices (caused by generation capacity limitations within the wheeling utility) can be calculated based on operating reserve margins. The loss coefficients (B matrix if a DC load flow is used) can be updated every hour if the utility has an on-line

1. Utility K's tie line spot prices do, in theory, depend to some extent on impedances, etc. of "nearby" lines of the interconnected network. However, various types of "equivalents for the outside world" can be used.

AC (DC) load flow or can be precomputed from off-line studies as is done in many utilities today. Computation of the effects of line flow and voltage limits on the spot prices requires sophisticated on line AC (or DC) load flow type capabilities although system operator judgement could possibly be used instead.

As with the tie line flow coefficients, the tie line bus spot prices can be calculated either on-line (1 hour or 24 hour update) or off line (TOU or flat). The off-line studies would require use of either Monte Carlo simulation or a combination of a probablistic production cost model with load flow analysis to capture expected patterns of demand, generation and transmission.

It is important to note that for many cases (especially if reactive energy can be ignored), the tie line flow coefficients (which require a global calculation involving all N utilities) are effectively constant unless major network reconfiguration occurs while the tie line bus spot prices (which are computed locally by each utility) can vary over wide ranges within a week or day. Thus 1 hour or 24 hour update wheeling rates can sometimes be calculated using 1 hour or 24 hour update tie line bus spot prices combined with TOU or flat tie line flow coefficients.

Revenue Reconciliation: The revenue reconciliation component of Eq. (5.1) can assume various forms depending on choice of approach (which is a policy decision). For Eq. (4.6), this component becomes specified after a value for the multiplier $\mu_R$ is fixed. Unless the wheeling utility is doing a lot of wheeling, this value will be determined primarily by conditions within the wheeling utility. Such multipliers could be specified a year in advance based on forecasted conditions or could be adapted to change, say each month, to follow changing conditions. Retroactive corrections to wheeling charges could also be done.

Transactions Costs: Wheeling imposes "transactions costs" on the wheeling utility because of the extra communication, computation, etc., required. Such transactions costs should be added to the wheeling rates. In some cases they can become large enough to prevent small wheeling transactions from occurring.

The level of sophistication that should be used in the specification and computation of wheeling rates depends on trade-offs between the transactions costs and the benefits achieved by having a good match of wheeling rates to costs. In the near term, the answer for any one region will depend heavily on what types of computer codes are presently implemented and what types of computing capacity is available. It should be emphasized that modification of computer programs presently operating in an on-line mode can be difficult.

## Section 6 Organization of Wheeling Markets

In Section 5 on-line (1 and 24 hour update) and off-line (time of use and flat) wheeling rates were discussed. Discussions now turn to three types of wheeling market structures which have on-line and off-line characteristics;

- o Spot Wheeling Market: On-Line
- o Long Term Wheeling Contracts: Off-Line
- o Futures Wheeling Market: Combines spot wheeling with long term contracts

### Three Utility Case, Single Transactions:

In a spot wheeling market for three utilities, Utility K posts wheeling rates (either 1 or 24 hour updates) and then Utilities B and S decide what, if any, type of transactions they want to conduct between themselves. If the amount of energy Utilities B and S want to transfer is large enough to affect Utility K's tie line bus spot prices, some iterations may be required or, alternately, Utility K posts wheeling rates as a function of the amount of wheeling energy involved.

In a long term wheeling contract market for the three utility case, Utility K sells wheeling rights, (kwh per hour) to Utilities B and S at a flat or time of use wheeling rate. The contract could be written such that Utilities B and S pay for these wheeling rights whether or not they use them.

A disadvantage of the simple long term wheeling contract is that it can be economically inefficient, in a global sense, at times when Utility K's wheeling costs are very high. For example, at hour 17 of the three bus example of Section 3, the total costs of Utilities K, B, and S could probably be reduced if wheeling did not occur. This problem can be overcome by combining long term wheeling contracts with a spot wheeling market to achieve a futures market in wheeling. In such a futures market, Utility K could sell Utilities B and S wheeling rights. However, the actual wheeling rates would be determined by the spot market values so that Utility K could "buy back the wheeling rights" at particular times when its actual wheeling costs exceed the rate fixed in the long term contract.

### N Utility Case: Multiple Transactions

For the general case, the same basic ideas still apply. Implementation is just more complex. One possible way to implement an N utility market is to establish a central wheeling coordination office (or wheeling broker) which has all of the necessary tie line flow coefficients stored either as constants, or as functions of time of day or overall demand. Each utility would then send its own 1 or 24 hour update tie line bus spot prices to the wheeling coordinating office, possibly expressed as a function of demand. Then if any two utilities want to consider a transaction, they can request from the office the necessary wheeling rates that all the rest of the utilities will charge. Many transactions can occur simultaneously because the wheeling rate is always on the margin. Of course, one transaction may effect the wheeling rate for others and vice versa.

If revenue reconciliation effects are sufficiently small and if all N utilities try to behave in an economically efficient manner to find the best possible buy/sell arrangements, the N utility spot wheeling market will tend to act like a centralized dispatch for all N utilities.

## Section 7 Comparison with Present Approaches

Present wheeling rates follow a variety of different approaches. Many are based on case specific negotiations and there is a lot of room for interpretation and "fair play" in implementation. The following illustrative features are taken primarily from Ref. [6].

- o The wheeling rates are flat and do not depend on the amount wheeled or other transmission flows. They often involve demand as well as energy charges.

- o They are often "postage stamp", i.e. the rate is the same regardless of distance wheeled. Voltage levels are usually a factor, however.

- o Some rates are for firm wheeling, i.e. Utility K can refuse to wheel only under emergency conditions. In these cases the rate is usually per peak kw wheeled during some period, or per contracted kw.

- o Rates are often determined by looking at embedded capital costs for some portion of Utility K's transmission system, with a measure of average losses added sometimes. One method is to draw a line on a map of the transmission system, from K's border with S to K's border with B.

- o Some arrangements allow Utility K to interrupt the wheeling at its sole discretion. In these cases the rates are often per kwh.

Present wheeling rates have evolved without the benefit of any method of calculating true wheeling costs, and during a period when wheeling was relatively infrequent. It is therefore not surprising to find major deviations from the optimal rates discussed in this paper. Major differences are:

- o The rates are not differentiated by the status of K's system, or by time (although tie line flow coefficients can be relatively constant, tie line spot prices are not).

- o Line and voltage limits are not explicitly included in cost calculations. Some arrangements give K the right to refuse to wheel, but this is inefficient since it is a 0-1 decision. If Utility B is willing to pay a high spot wheeling rate, it might be better for all parties, for K to re-dispatch its system to avoid the problem, (if possible) than for B to do without the wheeling.

- o Utility K can be made worse off by wheeling, because its gross wheeling revenue is not large enough to make up for its additional costs.

- o Utilities B and S can be made worse off if the wheeling rate is too high or if K refuses to wheel because of worries about inadequate revenues, or overloading its transmission system.

- o Rates are never negative. Therefore Utility K never encourages wheeling which would reduce its line losses or network constraints.

- o Under the present legal and financial system, Utilities S and B can often get wheeling merely by finding one utility, or a serial chain of utilities, between them which is willing to wheel. Other affected utilities are not necessarily compensated for the effects of the wheeling.

- o Without a firm basis for how to calculate rates, long elaborate quasi-judicial procedings are often needed to set wheeling rates.

## Section 8 Summary

We have presented a solid theoretical foundation for utility wheeling rates between investor owned utilities based on the principle that wheeling rates should be as close as possible to marginal costs, subject to revenue reconcilition. The computations required are non-trivial but are consistent with present day power system operations. Actual implementation can take many forms depending on trade offs between transaction costs and benefits.

When the wheeling rates of this paper are used, the buying and selling utilities automatically take the costs which they impose on other utilities into consideration when deciding whether to buy and sell to each other. Wheeling becomes a "no lose" proposition for all utilities.

The net effect of our approach is to allow closer coordination among spatially separated utilities. Economy interchange is feasible across long distances. The number of potential partners for interchanges increases, since they need not be members of the same pool nor adjacent to each other.

Areas for Further Study: Three areas which we hope to address in future papers and reports are:

- o Detailed tabulation of the different methods of revenue reconciliation and discussion of their implications. We only detailed one approach in this paper as an example.

- o Wheeling involving private parties and/or non-investor owned utilities. Our basic approach still applies but issues such as revenue reconciliation can change.

- o Incentives to build transmission facilities. Our approach makes the most efficient possible use of existing transmission facilities but longer run effects are tied to the treatment of revenue reconciliation, and to the spatial market power of individual wheeling utilities.

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## Appendix A Determination of Wheeling Rates Using the DC Load Flow Approximation

Wheeling rates are now derived explicitly determined in terms of power injections and network characteristics. The derivation assumes that the DC load flow approximation is reasonably accurate and reactive power and voltage magnitude constants can be disregarded. Define:

$\underline{B}^k$: $n^k \times n^k$ network losses matrix for Utility K. $n^k$ = total number of Utility K buses including intertie buses (i.e., tie line flows at the boundary of Utility K are considered injections at that point) but excluding Utility K's marginal generator bus which is selected as the swing bus. For a concise determination of this matrix in terms of network characteristics see Ref. [3].

$\underline{H}^k$: $m^k \times n^k$ DC flow matrix relating Utility K flows to injections. The swing bus is the bus of Utility K's marginal generator.

$\underline{T}^K(t)$: Vector of energy flows during hour t over tie lines of Utility K.

$\underline{P}^k(t)$, $\underline{Z}^k(t)$: Vectors of Utility K power injections and transmission line flows. Tie lines, tie line buses and the marginal generator bus are not included.

$\hat{\underline{Z}}^k(t)$: $m^k \times 1$ vector of Utility K real power line flows. Tie lines are included with a bus added at their end. Power injections at these buses (positive or negative) are set equal to tie line flows. Contains $\underline{T}^K(t)$ and $\underline{Z}^k(t)$.

$\hat{\underline{P}}^k(t)$: $n^k \times 1$ vector of real power injections at each bus of Utility K including the buses at the end of tie lines. The marginal generator bus of Utilitiy K is not included as it is selected as the swing bus. Contains $\underline{T}^K(t)$ and $\underline{P}^k(t)$.

$\underline{H}(b)$: $m \times n$ DC load flow matrix for whole system of interconnected utilities. n is the number of all buses in all utilities except for the bus of Utility B's marginal generator which is selected as the swing bus. There are N such matrices (one for each buying utility). They change with time to reflect changes in transmission line availabilities and marginal generator locations. For a concise derivation of this matrix in terms of network characteristics see Ref. [3].

$s^*$: Index corresponding to the bus of the selling utility's margnial generator.

$\underline{h}(b)^{s*k}$: Vector with same dimension as $\underline{T}^K(t)$. It is made up of elements of $\underline{H}(b)$ in column $s^*$ and rows corresponding to Utility K's tie lines.

$\underline{P}(t)$, $\underline{Z}(t)$: Vectors of system wide (all N utilities) injections and flows respectively. Bus of Utility B's marginal generator is not included since it is selected as the system wide swing bus.

Using the above definitions and dropping time arguments for simplicity we note:

a) Flows are related to injections by the relationship

$$\underline{Z} = H(b)\ \underline{P} \quad \text{and hence} \quad \frac{\partial T^K}{\partial \underline{P}_{s*}} = \underline{h}(b)^{s*k}$$

b) The condition $\frac{\partial I^S}{\partial I^B} = -1$ is built in the construction of H(b) by virtue of the selection of Utility B's marginal generator location as the swing bus.

c) Economic unit dispatch and the lossless model of DC load flow approximation imply

$$\partial I^S = \partial P_{s*} \quad \text{and} \quad \partial I^B = \partial P_{b*}$$

where $P_{s*}$ is the element of $\underline{P}$ corresponding to the selling utility's marginal generator, and $P_{b*}$ is similarly defined for the buying utility.

d) Utility K real power spot prices at tie line buses, $\underline{\rho}^K$ are those elements of the vector shown below which corresponds to buses with injections $\underline{T}^K$ (i.e., buses at the end of tie lines)

$$(\lambda_k + \mu_k)\ (1 - 2\underline{B}\ \hat{\underline{P}}^k) + \underline{H}^{k'}\underline{\eta}_k$$

where $\lambda_k$ is the spot price at Utility K's marginal generator bus (i.e. system lambda), and $\mu_k$, $\underline{\eta}_k$ are Lagrange multipliers whose values are selected to yield energy balance and observe line flow constaints (see Ref. [2]).

Thus, the optimal wheeling rates when the DC load flow approximation is adopted can be explicitly calculated as

$$\omega_{K,BS}(t) = \underline{\rho}^K(t)\underline{h}(b)^{s*k}$$

where $\underline{\rho}^K(t)$ and $h(b)^{s*k}$ are obtained as outlined above.

## Appendix B Derivation of Revenue Reconciliation Formula

A simple formula, Eq. (4.6) was given in the main text as an example of one reasonable approach to revenue reconciliation. More general discussions are now given which yield Eq. (4.6) as a special case.

One general approach is to define the customer rates $r_j(t)$ and wheeling rates $\omega(t)$ to be "as close as possible" to the $r_j^*(t)$ and $\omega^*(t)$ (their values without revenue reconciliation) subject to the revenue reconciliation constraint with a weighted least squares criteria. Thus $r_j(t)$ and $\omega(t)$ are defined as those quantities that minimize the Lagrangian

$$\mathcal{L} = \sum_{t=1}^{8760} \{\sum_j [r_j(t) - r_j^*(t)]^2\ Q_j(t)] + [\omega(t) - \omega^*(t)]^2\ Q_w(t)\} + 2\mu_R]R_T - \sum_{t=1}^{8760}[\sum_j r_j(t)\ d_j(t) + \omega(t)\ W(t)]] \qquad (B.1)$$

$Q_j(t)$, $Q_w(t)$: Positive weighting functions.

$\mu_R$: Lagrange multiplier determined by revenue reconcilation constraint. May be positive or negative.

Setting $\partial\mathcal{L}/\partial\omega = 0$ yields

$$\omega(t) = \omega^*(t) + \mu_R\ Q_w^{-1}(t)\ W(t) \qquad (B.2)$$

with a similar equation for $r_j(t)$. It follows from Eq. (B.1) that when $Q_w(t)$ is large, $\omega(t)$ tends to be closer to $\omega^*(t)$ than when $Q_w(t)$ is small. One reasonable way to choose $Q_w(t)$ is to make $\omega(t)$ closest to $\omega^*(t)$ when

- o The transaction $W(t)$ is large and/or
- o The magnitude of the wheeling rate $\omega^*(t)$ is small.

One $Q_w(t)$ which satisfies this criteria is

$$Q_w(t) = \frac{W(t)}{|\omega^*(t)|} \qquad (B.3)$$

Substituting Eq. (B.3) into Eq. (B.2) yields Eq. (4.6), i.e.,

$$\omega(t) = \omega^*(t) + \mu_R\ |\omega^*(t)|$$

Several other possible approaches are

$$Q_w(t) = 1 \Rightarrow \omega(t) = \omega^*(t) + \mu_R W(t)$$
$$Q_w(t) = W(t) \Rightarrow \omega(t) = \omega^*(t) + \mu_R \qquad (B.4)$$
$$Q_w(t) = |\omega^*(t)|^{-1} \Rightarrow \omega(t) = \omega^*(t) + \mu_R W(t)\,|\omega^*(t)|$$

In economic theory, a more common approach to revenue reconciliation is to define a social welfare cost function (utility costs minus benefits to customers) which is minimized subject to the revenue constraint assuming all customers behave in an optimum fashion. This approach yields "second best" and "Ramsey type" pricing formulas which have the "inverse elasticity" property. Unfortunately these results depend on price elasticities which can be difficult to specify. The social welfare equations can behave very similarly or quite differently than the weighted least squares equations depending on what price elasticities are assumed.